San Diego Math League High School Division, Round 1a
November 15, 2008

---

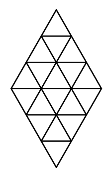
1. If p is chosen from the set $\{1,3,5\}$ and q is chosen from the set $\{2,4,6,8\}$, then the number of ways that p and q can be chosen so that $p+q \leq 10$ is

   (A) 7 (B) 8 (C) 9 (D) 10 (E) 12

2. The value of 1/3 of 6^{30} is

   (A) 6^{10} (B) 2^{30} (C) 2^{10} (D) 2×6^{29} (E) 2×6^{10}

3. In the diagram, all triangles are equilateral.



   The total number of equilateral triangles of any size is

   (A) 18 (B) 20 (C) 24 (D) 26 (E) 28

4. An arithmetic progression with positive common difference contains the numbers 13, 25 and 41. Which of the following numbers is also in the progression?

   (A) 2008 (B) 1990 (C) 2003 (D) 1993 (E) 1998

5. After a wooden $4 \times 5 \times 6$ block is completely painted green, it is cut into $1 \times 1 \times 1$ cubes. What is the probability that a randomly chosen $1 \times 1 \times 1$ cube has exactly one green face?

   (A) $\dfrac{47}{60}$ (B) $\dfrac{13}{60}$ (C) $\dfrac{47}{120}$ (D) $\dfrac{13}{30}$ (E) $\dfrac{1}{5}$

6. Side $\overline{MP}$ of parallelogram $MNOP$ is extended through P to point R such that P is the midpoint of the segment $\overline{MR}$. $\overline{NR}$ intersects diagonal $\overline{MO}$ at Q and side $\overline{PO}$ at S. If $RS = 30$, then what is SQ?

   (A) $4\sqrt{5}$ (B) 12 (C) 10 (D) 8 (E) $5\sqrt{3}$

7. In how many zeroes does the number 2008! end?

   (A) 450 (B) 500 (C) 200 (D) 445 (E) 600

8. If

$$\frac{30}{7} = x + \cfrac{1}{y + \cfrac{1}{z}},$$

   where x, y, and z are positive integers, then what is the value of $x+y+z$?

   (A) 9 (B) 11 (C) 13 (D) 30 (E) 37

9. What is the value of the sum $\dfrac{1}{\log_2 36} + \dfrac{1}{\log_3 36} + \dfrac{1}{\log_6 36}$?

   (A) $\dfrac{1}{2}$ (B) $\dfrac{3}{4}$ (C) $\log_{36} 11$ (D) 1 (E) $\log_{11} 36$

10. A quadrilateral inscribed in a circle has sides 7, 24, 15, and 20, in that order. What is the quadrilateral's area?

    (A) 234 (B) 123 (C) 25 (D) 187 (E) 100

11. In how many ways can 12 boys be divided into 3 teams of 4 boys each?

    (A) 720 (B) 4225 (C) 5775 (D) 166325 (E) 5025

12. Let ABC be a right-angled triangle, such that the measure of $\angle C$ is 30 degrees less than the larger of $m\angle A$ and $m\angle B$. Let R be its circumradius and let r be its inradius. What is $\frac{R}{r}$?

    (A) $\sqrt{3}$ (B) 2 (C) $\sqrt{3}+1$ (D) $\sqrt{3}-1$ (E) $1+\frac{\sqrt{3}}{2}$

13. How many pairs of positive integers (a, b), in which a and b are both less than 100, satisfy

$$a\sqrt{2a+b} = b\sqrt{b-a}?$$

    (A) 25 (B) 0 (C) 75 (D) 49 (E) 51

14. Which of the following is equal to $(17+2\sqrt{52})^{3/2} - (17-2\sqrt{52})^{3/2}$?

    (A) 155 (B) 172 (C) 200 (D) 209 (E) 320

15. Suppose that $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = 1$. Find $\cos(\alpha+\beta)$.

    (A) $\frac{5}{13}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1-\sqrt{2}}{2}$ (E) $\frac{3}{7}$

San Diego Math League High School Division, Round 1b
November 15, 2008

1. Points A, B, C, and D lie on a line, in that order. If $AB : BC = 1 : 2$ and $BC : CD = 8 : 5$, then find $AB : BD$.

2. Four standard 6-sided dice are rolled. What is the probability that exactly three of the dice show a number larger than 4?

3. Three congruent circles of radius 1 are given such that each one passes through the centers of the other two. Find the area of region common to all three circles.

4. A function $f(x)$ is such that we have
$$f(1-x) + (1-x)f(x) = 4$$
for all real x. Find $f(4)$.

5. Find the total number of integer pairs (a, b) that satisfy the equation
$$9a^2 + 3b = b^2 + 8.$$

6. Find the sum of the coefficients of x^{17} and x^{18} in the expansion of $(1 + x^5 + x^7)^{20}$.

7. Find the largest integer n such that n is divisible by all positive integers smaller than $\sqrt{n}$.

8. Two hunters, Bruce and Louise, set out to hunt ducks. Each of them hits as often as he or she misses when shooting at ducks. Bruce shoots at 7 ducks during the hunt, while Louise shoots at 9. Let p be the probability that Louise hits more ducks than Bruce and let q be the probability that Bruce hits at least as many ducks as Louise. If $\frac{a}{2^b} = p - q$, where a is odd, then find $a + b$.